Update of the stranger story: The strange vector form factors of the nucleon in the SU(3) chiral quark-soliton model

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Abstract

The strange vector form factors are evaluated for $Q^2=0$ and $Q^2=1~{\rm GeV}^2$ in the framework of the SU(3) chiral quark-soliton model (or semi-bosonized SU(3) Nambu-Jona-Lasinio model). The rotational $1/N_c$ and m_s corrections are taken into account up to linear order. The mean-square strange radius $\langle r^2 \rangle_s^{Sachs} = -0.35~{\rm fm}^2$ and the strange magnetic moment $\mu_s = -0.44~\mu_N$ are obtained. The results are compared with several different models.

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I. INTRODUCTION

The strangeness content of the nucleon has been under a great deal of discussions for well over a decade. A few years ago, the European Muon Collaboration (EMC) [1] measured the spin structure function of the proton in deep inelastic muon scattering and showed that there is an indication of a sizable strange quark contribution. This remarkable result has been confirmed by following experiments of the Spin Muon Collaboration (SMC) [2,3], E142 and E143 collaborations [4,5].

Another experiment conducted at Brookhaven [6] (BNL experiment 734) measuring the low-energy elastic neutrino-proton scattering came to the more or less same conclusion. Kaplan and Manohar [7] showed how elastic νp and ep scatterings can be used to extract not only the G_1 form factors of the $U(1)_A$ current but also the F_2 form factors of the baryon number current and furthermore how the strange quark matrix elements $\langle p|\bar{s}\gamma_{\mu}\gamma_5 s|p\rangle$ and $\langle p|\bar{s}\gamma_{\mu}s|p\rangle$ can be evaluated. Following these suggestions, Garvey et al. [8] reanalyzed the above-mentioned νp elastic scattering experiment and determined proton strange form factors in particular at $Q^2=0$, pointing out the shortcomings of the analysis done by Ref. [6] . The best fit of Ref. [8] with the smallest χ^2 tells $F_1^s=0.53\pm0.70$ and $F_2^s=-0.40\pm0.72$. By comparing the different Q^2 dependence of $d\sigma/dQ^2(\nu p)$ to $d\sigma/dQ^2(\bar{\nu}p)$, Garvey et al favor $F_1^s(Q^2)>0$ and $F_2^s(Q^2)<0$. However, these form factors are experimentally unknown to date and have no stringent and concrete constraints on their Q^2 -dependence yet. There are various proposals and experiments in progress (see Refs. [19,20] for details). All these considerations lead to the conclusion that, in contrast to the naive quark model, it is of great importance to consider strange quarks in the nucleon seriously.

There have been several theoretical efforts to describe the strange form factors of the nucleon. The first attempt was performed by Jaffe [9]. Jaffe took advantage of Ref. [10], i.e. the pole fit analysis based on dispersion theory and estimated the mean-square strange radius and magnetic moment of the nucleon: $\langle r^2 \rangle_s^{Dirac} = 0.16 \pm 0.06 \text{ fm}^2$, $\mu_s = -0.31 \pm 0.09 \mu_N$. More recently, Hammer et al. [11] updated Jaffe's pole-fit analysis of the strange vector form

factors, relying upon a new dispersion theoretic analysis of the nucleon electromagnetic form factors. In fact, Hammer *et al.* improved Jaffe's prediction, giving $\mu_s = -0.24 \pm 0.03 \ \mu_N$ and $\langle r^2 \rangle_s^{Dirac} = 0.21 \pm 0.03 \ \text{fm}^2$. A noticeable point of the pole-fit analysis is that it has the different sign of the strange electric radius, compared with almost other models.

Another interesting approach is the kaon-loop calculation. The main idea of the kaon-loop calculation is that the strangeness content of the nucleon exists as a pair of $K\Lambda$ or $K\Sigma$ components. Koepf *et al.* [12] first evaluated μ_s and $\langle r^2 \rangle_s^{Dirac}$, considering the possible kaon loops relevant for the strange vector form factors. However, Ref. [12] failed to include seagull terms which are essential to satisfy the Ward-Takahashi identity in the vector current sector. Musolf *et al.* [13] added these seagull terms and obtained $\mu_s = -(0.31 \to 0.40) \ \mu_N$ and $\langle r^2 \rangle_s^{Dirac} = -(6.68 \to 6.90) \times 10^{-3} \ \text{fm}^2$. The prediction of $\langle r^2 \rangle_s^{Dirac}$ in the kaon-loop calculation is found to be much smaller than the pole-fit analysis. To reconcile the conflict between the pole-fit analysis and the kaon-loop calculation, Refs. [14,18] suggested the combination of the vector meson dominance (VMD) and $\omega - \phi$ mixing in the vector-isovector channel with the kaon-loop calculation. The value of $\langle r^2 \rangle_s^{Dirac}$ in Ref. [14] appeared to be larger than that of the kaon-loop calculation but still conspicuosly smaller than that of the pole-fit analysis: $\langle r^2 \rangle_s^{Dirac} = -(2.42 \to 2.45) \times 10^{-2} \ \text{fm}^2$. Ref. [18] evaluated also the strange vector form factors and discussed to a great extent several different theoretical estimates.

The SU(3) Skyrme model with pseudoscalar mesons [15] and with vector mesons [16] estimated, respectively, $\mu_s = -0.13$, $\mu_s = -0.05$ and $\langle r^2 \rangle_s^{Dirac} = -0.10$ fm², $\langle r^2 \rangle_s^{Dirac} = 0.05$ fm². Most recently, Leinweber obtained $\mu_s = -0.75 \pm 0.30$ μ_N which appears to be much larger than other models.

In this paper, we aim at investigating the strange vector form factors and related strange observables in the SU(3) chiral quark-soliton model (χ QSM), often called semi-bosonized SU(3) Nambu-Jona-Lasinio model (NJL). The model is based on the interaction of quarks with Goldstone bosons and has been shown to be quite successful in reproducing static properties of the baryons such as mass splitting [21,22], axial constants [23] and magnetic moments [24] and their form factors [25,26]. In a recent review [27], one can easily see how

well the model describe the baryonic observables. In particular, since the strange vector form factors are deeply related to the electromagnetic form factors [9,11] being well described in χ QSM, it is quite interesting to study them in the same framework.

The outline of the paper is as follows: Section II develops the general formalism for obtaining the strange vector form factors in the framework of χ QSM. Section III presents the corresponding results and discuss them. Section IV contains a summary and draws the conclusion of the present work.

II. GENERAL FORMALISM

In this section we briefly review the formalism of χ QSM. Details can be found in ref. [27]. We start with the low-energy partition function in Euclidean space given by the functional integral over pseudoscalar meson (π^a) and quark fields(ψ):

$$\mathcal{Z} = \int \mathcal{D}\psi \mathcal{D}\psi^{\dagger} \mathcal{D}\pi^{a} \exp\left(-\int d^{4}x \psi^{\dagger} i D\psi\right),
= \int \mathcal{D}\pi^{a} \exp\left(-S_{eff}[\pi]\right),$$
(1)

where S_{eff} is the effective action

$$S_{eff}[\pi] = -\text{Spln}iD.$$
 (2)

iD represents the Dirac differential operator

$$iD = \beta(-i\partial \!\!\!/ + \hat{m} + MU^{\gamma 5}) \tag{3}$$

with the pseudoscalar chiral field

$$U^{\gamma_5} = \exp(i\pi^a \lambda^a \gamma_5) = \frac{1+\gamma_5}{2} U + \frac{1-\gamma_5}{2} U^{\dagger}. \tag{4}$$

 \hat{m} is the matrix of the current quark mass given by

$$\hat{m} = \operatorname{diag}(m_u, m_d, m_s) = m_0 \mathbf{1} + m_8 \lambda_8, \tag{5}$$

where λ^a designate the usual Gell-Mann matrices normalized as $\operatorname{tr}(\lambda^a\lambda^b)=2\delta^{ab}$. Here, we have assumed isospin symmetry $(m_u=m_d)$. M stands for the dynamical quark mass arising from the spontaneous chiral symmetry breaking, which is in general momentum-dependent [28]. We regard M as a constant and introduce the proper-time regularization for convenience. The m_0 and m_8 in Eq. (5) are defined, respectively, by

$$m_0 = \frac{m_u + m_d + m_s}{3}, \quad m_8 = \frac{m_u + m_d - 2m_s}{2\sqrt{3}}.$$
 (6)

The operator iD is expressed in Euclidean space in terms of the Euclidean time derivative ∂_{τ} and the Dirac one–particle Hamiltonian $H(U^{\gamma_5})$

$$iD = \partial_{\tau} + H(U^{\gamma_5}) + \beta \hat{m} - \beta \bar{m} \mathbf{1} \tag{7}$$

with

$$H(U^{\gamma_5}) = \frac{\vec{\alpha} \cdot \nabla}{i} + \beta M U^{\gamma_5} + \beta \bar{m} \mathbf{1}. \tag{8}$$

 \bar{m} is defined by $(m_u + m_d)/2 = m_u = m_d$. β and $\vec{\alpha}$ are the well–known Dirac Hermitian matrices. The U is assumed to have a structure corresponding to the so-called trivial embedding of the SU(2)-hedgehog into SU(3):

$$U = \begin{pmatrix} U_0 & 0 \\ 0 & 1 \end{pmatrix}, \tag{9}$$

with

$$U_0 = \exp\left[i\vec{n}\cdot\vec{\tau}P(r)\right]. \tag{10}$$

The profile function P(r) is determined numerically by solving the Euler-Lagrange equation corresponding to $\frac{\delta S_{eff}}{\delta P(r)} = 0$. This yields a selfconsistent classical field U_0 and a set of single quark energies and corresponding states E_n and Ψ_n . Note that the E_n and Ψ_n do not constitute the nucleon $|N\rangle$ yet because the collective spin and and isospin quantum numbers are missing. Those are obtained by the semiclassical quantization procedure, described below in the context of the strange form factors.

The information of the strange vector form factors in the nucleon is contained in the quark matrix elements as follows:

$$\langle N'(p')|J_{\mu}^{s}|N(p)\rangle = \langle N'(p')|\bar{s}\gamma_{\mu}s|N(p)\rangle. \tag{11}$$

The strange Dirac form factors of the nucleon are defined by the matrix elements of the J^s_{μ} :

$$\langle N'(p')|J_{\mu}^{s}|N(p)\rangle = \bar{u}_{N}(p')\left[\gamma_{\mu}F_{1}^{s}(q^{2}) + i\sigma_{\mu\nu}\frac{q^{\nu}}{2M_{N}}F_{2}^{s}(q^{2})\right]u_{N}(p),$$
 (12)

where q^2 is the square of the four momentum transfer $q^2=-Q^2$ with $Q^2>0$. M_N and $u_N(p)$ stand for the nucleon mass and its spinor, respectively. The strange quark current J^s_μ can be expressed in terms of the baryon current and the hypercharge current:

$$J_{\mu}^{s} = \bar{s}\gamma_{\mu}s = J_{\mu}^{B} - J_{\mu}^{Y} = \bar{q}\gamma_{\mu}\hat{Q}_{s}q,$$
 (13)

where

$$J_{\mu}^{B} = \frac{1}{N_{c}} \bar{q} \gamma_{\mu} q,$$

$$J_{\mu}^{Y} = \frac{1}{\sqrt{3}} \bar{q} \gamma_{\mu} \lambda_{8} q$$

$$\hat{Q}_{s} = \frac{1}{N_{c}} - \frac{1}{\sqrt{3}} \lambda_{8},$$
(14)

where N_c denotes the number of colors of the quark. $\hat{Q}_s = \text{diag}(0, 0, 1)$ is called *strangeness* operator. We employ the non-standard sign convention used by Jaffe [9] for the strange current. The baryon and hypercharge currents are equal to the singlet and octet currents, respectively.

The strange Dirac form factors F_1^s and F_2^s can be written in terms of the strange Sachs form factors, $G_E^s(Q^2)$ and $G_M^s(Q^2)$:

$$G_E^s(Q^2) = F_1^s(Q^2) - \frac{Q^2}{4M_N^2} F_2^s(Q^2)$$

$$G_M^s(Q^2) = F_1^s(Q^2) + F_2^s(Q^2). \tag{15}$$

In the non-relativistic limit $(Q^2 \ll M_N^2)$, the Sachs-type form factors $G_E^s(Q^2)$ and $G_M^s(Q^2)$ are related to the time and space components of the strange current, respectively:

$$\langle N'(p')|J_0^s(0)|N(p)\rangle = G_E^s(Q^2)$$

$$\langle N'(p')|J_i^s(0)|N(p)\rangle = \frac{1}{2M_N}G_M^s(Q^2)i\epsilon_{ijk}q^j\langle\lambda'|\sigma_k|\lambda\rangle,$$
(16)

where σ_k stand for Pauli spin matrices. The $|\lambda\rangle$ is the corresponding spin state of the nucleon. The matrix elements of the strange quark current can be related to a correlator:

$$\langle N'(p')|\bar{s}\gamma_{\mu}s|N(p)\rangle_{T\to\infty} \langle 0|J_{N'}(\vec{x},T/2)\bar{q}\gamma_{\mu}\hat{Q}_sqJ_N^{\dagger}(\vec{y},-T/2)|0\rangle. \tag{17}$$

The nucleon current J_N can be built from N_c quark fields

$$J_N(x) = \frac{1}{N_c!} \epsilon_{i_1 \cdots i_{N_c}} \Gamma_{JJ_3 TT_3 Y}^{\alpha_1 \cdots \alpha_{N_c}} \psi_{\alpha_1 i_1}(x) \cdots \psi_{\alpha_{N_c} i_{N_c}}(x). \tag{18}$$

 $\alpha_1 \cdots \alpha_{N_c}$ denote spin–flavor indices, while $i_1 \cdots i_{N_c}$ designate color indices. The matrices $\Gamma_{JJ_3TT_3Y}^{\alpha_1\cdots\alpha_{N_c}}$ are taken to endow the corresponding current with the quantum numbers JJ_3TT_3Y . In our model, Eq. (17) is represented by the Euclidean functional integral with regards to quark and pseudo-Goldstone fields:

$$\langle N'(p')|\bar{q}\gamma_{\mu}\hat{Q}_{s}q|N(p)\rangle = \frac{1}{\mathcal{Z}}\lim_{T\to\infty}\exp\left(ip_{4}\frac{T}{2} - ip'_{4}\frac{T}{2}\right)$$

$$\times \int d^{3}xd^{3}y\exp\left(-i\vec{p'}\cdot\vec{y} + i\vec{p}\cdot\vec{x}\right)\int \mathcal{D}U\int \mathcal{D}\psi\int \mathcal{D}\psi^{\dagger}$$

$$\times J_{N'}(\vec{y},T/2)q^{\dagger}(0)\beta\gamma_{\mu}\hat{Q}_{s}q(0)J_{N}^{\dagger}(\vec{x},-T/2)$$

$$\times \exp\left[-\int d^{4}z\psi^{\dagger}iD\psi\right],$$
(19)

where \mathcal{Z} stands for the normalization factor which is expressed by the same functional integral but without the quark current operator $\bar{s}\gamma_{\mu}s$. Since \bar{m} is much smaller than m_s , we use $\hat{m} - \bar{m}\mathbf{1} \simeq diag(0, 0, m_s)$ in the perturbation. Eq.(19) can be decomposed into valence and sea contributions:

$$\langle N'(p')|\bar{q}\gamma_{\mu}\hat{Q}_{s}q|N(p)\rangle = \langle N'(p')|\bar{q}\gamma_{\mu}\hat{Q}_{s}q|N(p)\rangle_{val} + \langle N'(p')|\bar{q}\gamma_{\mu}\hat{Q}_{s}q|N(p)\rangle_{sea}, \qquad (20)$$

where

$$\langle N'(p')|V_{\mu}(0)|N(p)\rangle_{val} = \frac{1}{\mathcal{Z}}\Gamma_{J'J'_{3}T'T'_{3}Y'}^{\beta_{1}\cdots\beta_{N_{c}}}\Gamma_{JJ_{3}TT_{3}Y}^{\alpha_{1}\cdots\alpha_{N_{c}}*}\lim_{T\to\infty}\exp\left(ip_{4}\frac{T}{2}-ip_{4}'\frac{T}{2}\right)$$

$$\times \int d^{3}x d^{3}y \exp\left(-i\vec{p'}\cdot\vec{y}+i\vec{p}\cdot\vec{x}\right)$$

$$\times \int \mathcal{D}U \exp\left(-S_{eff}\right) \sum_{i=1}^{N_{c}} \beta_{i} \langle \vec{y}, T/2 | \frac{1}{iD} | 0, t_{z} \rangle_{\gamma} [\beta \gamma_{\mu} \hat{Q}_{s}]_{\gamma\gamma'}$$

$$\times \gamma' \langle 0, t_{z} | \frac{1}{iD} | \vec{x}, -T/2 \rangle_{\alpha_{i}} \prod_{j \neq i}^{N_{c}} \beta_{j} \langle \vec{y}, T/2 | \frac{1}{iD} | \vec{x}, -T/2 \rangle_{\alpha_{j}} \tag{21}$$

and

$$\langle N'(p')|V_{\mu}(0)|N(p)\rangle_{sea} = -\frac{N_c}{\mathcal{Z}} \Gamma_{J'J'_3T'T'_3Y'}^{\beta_1\cdots\beta_{N_c}} \Gamma_{JJ_3TT_3Y}^{\alpha_1\cdots\alpha_{N_c}*} \lim_{T\to\infty} \exp\left(ip_4 \frac{T}{2} - ip'_4 \frac{T}{2}\right)$$

$$\times \int d^3x d^3y \exp\left(-i\vec{p'}\cdot\vec{y} + i\vec{p}\cdot\vec{x}\right)$$

$$\times \int \mathcal{D}U \exp\left(-S_{eff}\right) \operatorname{Tr}_{\gamma\lambda} \langle 0, t_z| \frac{1}{iD} [\beta\gamma_{\mu}] \hat{Q}_s |0, t_z\rangle$$

$$\times \prod_{i=1}^{N_c} \beta_i \langle \vec{y}, T/2| \frac{1}{iD} |\vec{x}, -T/2\rangle_{\alpha_i}.$$

$$(22)$$

 S_{eff} is the effective chiral action expressed by

$$S_{eff} = -N_c \operatorname{Spln} \left[\partial_{\tau} + H(U^{\gamma_5}) + \beta \hat{m} - \beta \bar{m} \mathbf{1} \right]. \tag{23}$$

In order to perform the collective quantization, we have to integrate Eqs. (21) and (22) over small oscillations of the pseudo-Goldstone field around the saddle point Eq. (9). This will not be done except for the zero modes. The corresponding fluctuations of the pion fields are not small and hence cannot be neglected. The zero modes are relevant to continuous symmetries in our problem. In particular, we have to take into account the translational zero modes properly in order to evaluate form factors, since the soliton is not invariant under translation and its translational invariance is restored only after integrating over the translational zero modes. Explicitly, the zero modes are taken into account by considering a slowly rotating and translating hedgehog:

$$\tilde{U}(\vec{x},t) = A(t)U(\vec{x} - \vec{Z}(t))A^{\dagger}(t). \tag{24}$$

A(t) belongs to an SU(3) unitary matrix. The Dirac operator $i\tilde{D}$ in Eq. (7) can be written as

$$i\tilde{D} = \left(\partial_{\tau} + H(U^{\gamma_5}) + A^{\dagger}(t)\dot{A}(t) - i\beta\dot{Z}\cdot\nabla + \beta A^{\dagger}(t)(\hat{m} - \bar{m}\mathbf{1})A(t)\right). \tag{25}$$

The corresponding collective action is expressed by

$$\tilde{S}_{eff} = -N_c \operatorname{Sp} \ln \left[\partial_{\tau} + H(U^{\gamma_5}) + A^{\dagger}(t)\dot{A}(t) - i\beta \dot{\vec{Z}} \cdot \nabla \right]
+ \beta A^{\dagger}(t)(\hat{m} - \bar{m}\mathbf{1})A(t) - \beta A^{\dagger}(t)s_{\mu}\gamma_{\mu}\hat{Q}_s A(t) \tag{26}$$

with the angular velocity

$$A^{\dagger}(t)\dot{A}(t) = i\Omega_E = \frac{1}{2}i\Omega_E^a \lambda^a \tag{27}$$

and the velocity of the translational motion

$$\dot{\vec{Z}} = \frac{d}{dt}\vec{Z}.$$
 (28)

Hence, Eq. (21) and Eq. (22) can be written in terms of the rotated Dirac operator $i\tilde{D}$ and chiral effective action \tilde{S}_{eff} . The functional integral over the pseudoscalar field U is replaced by the path integral which can be calculated in terms of the eigenstates of the Hamiltonian corresponding to the collective action and these Hamiltonians can be diagonalized in an exact manner.

We take into account the rotational $1/N_c$ corrections and m_s corrections up to linear order:

$$\frac{1}{i\tilde{D}} \simeq \frac{1}{\partial_{\tau} + H} + \frac{1}{\partial_{\tau} + H} (-i\Omega_E) \frac{1}{\partial_{\tau} + H} + \frac{1}{\partial_{\tau} + H} (-\beta A^{\dagger} [\hat{m} - \bar{m} \mathbf{1}] A) \frac{1}{\partial_{\tau} + H}. \tag{29}$$

When the m_s corrections are considered, SU(3) symmetry is no more exact. Thus, the eigenfunctions of the collective Hamiltonian are neither in a pure octet nor in a pure decuplet but in mixed states with higher representations:

$$|8, N\rangle = |8, N\rangle + c_{\bar{10}}|\bar{10}, N\rangle + c_{27}|27, N\rangle$$
 (30)

with

$$c_{\bar{10}} = \frac{\sqrt{5}}{15}(\sigma - r_1)I_2m_s, \quad c_{27} = \frac{\sqrt{6}}{75}(3\sigma + r_1 - 4r_2)I_2m_s.$$
 (31)

The constant σ is related to the SU(2) πN sigma term $\Sigma_{SU(2)} = 3/2(m_u + m_d)\sigma$ and r_i designates K_i/I_i , where K_i stand for the anomalous moments of inertia defined in Ref. [21].

Having carried out a lengthy manipulation (for details, see Ref. [27]), we arrive at our final expressions for the strange vector form factors. The Sachs strange electric form factor G_E^s is expressed as follows (see appendix A for detail):

$$G_{E}^{s}(\vec{Q}^{2}) = (1 - \langle D_{88}^{(8)} \rangle_{N}) \mathcal{B}(Q^{2})$$

$$+ \langle D_{8a}^{(8)} J_{a} \rangle_{N} \frac{2\mathcal{I}_{1}(Q^{2})}{\sqrt{3}I_{1}} + \langle D_{8p}^{(8)} J_{p} \rangle_{N} \frac{2\mathcal{I}_{2}(Q^{2})}{\sqrt{3}I_{2}}$$

$$+ (1 - \langle D_{88}^{(8)} \rangle_{N}) m_{s} \mathcal{C}(Q^{2})$$

$$+ \langle D_{8a}^{(8)} D_{8a}^{(8)} \rangle_{N} \frac{4m_{s}}{3I_{1}} \left(I_{1} \mathcal{K}_{1}(\vec{Q}^{2}) - \mathcal{I}_{1}(\vec{Q}^{2}) \mathcal{K}_{1} \right)$$

$$+ \langle D_{8p}^{(8)} D_{8p}^{(8)} \rangle_{N} \frac{4m_{s}}{3I_{2}} \left(I_{2} \mathcal{K}_{2}(\vec{Q}^{2}) - \mathcal{I}_{2}(\vec{Q}^{2}) \mathcal{K}_{2} \right),$$

$$(32)$$

 I_i and K_i are the moments of inertia and anomalous moments of inertia [21], respectively, \mathcal{B} , \mathcal{I}_i , and \mathcal{K}_i correspond to the baryon number, moments of inertia, and the anomalous moments of inertia at $Q^2 = 0$, respectively. From Eq.(32), we can see easily that at $Q^2 = 0$ the strange electric form factor G_E^s vanishes (note that $\mathcal{C}(Q^2 = 0) = 0$). Making use of the relation $\sum_{a=1}^8 D_{8a}^{(8)} J_a = -\sqrt{3}Y/2$ and $J_8 = -N_c/(2\sqrt{3})$, we obtain $G_E^s(Q^2 = 0) = B - Y = S$. Since the net strangeness of the nucleon is zero, G_E^s at $Q^2 = 0$ must vanish. The final expression of the Sachs strange magnetic form factor is written (see appendix A for detail) by

$$G_{M}^{s}(\vec{Q}^{2}) = \frac{M_{N}}{|\vec{Q}|} \left[-\frac{\langle D_{83}^{(8)} \rangle_{N}}{\sqrt{3}} \left(\mathcal{Q}_{0}(\vec{Q}^{2}) + \frac{\mathcal{Q}_{1}(\vec{Q}^{2})}{I_{1}} + \frac{\mathcal{Q}_{2}(\vec{Q}^{2})}{I_{2}} \right) \right.$$

$$+ \left. \langle (D_{88}^{(8)} - 1)J_{3} \rangle_{N} \frac{\mathcal{X}_{1}(\vec{Q}^{2})}{3I_{1}} + \left. \langle d_{3pq}D_{8p}^{(8)}J_{q} \rangle_{N} \delta_{pq} \frac{\mathcal{X}_{2}(\vec{Q}^{2})}{\sqrt{3}I_{2}} \right.$$

$$- \frac{2m_{s}}{\sqrt{3}} \langle (D_{88}^{(8)} - 1)D_{83}^{(8)} \rangle_{N} \mathcal{M}_{0}(\vec{Q}^{2})$$

$$+ \frac{m_{s}}{3} \langle D_{83}^{(8)} \rangle_{N} \left(2\mathcal{M}_{1}(\vec{Q}^{2}) - \frac{2}{\sqrt{3}} r_{1} \mathcal{X}_{1}(\vec{Q}^{2}) \right)$$

$$- \frac{m_{s}}{\sqrt{3}} \langle D_{83}^{(8)} D_{88}^{(8)} \rangle_{N} \left(2\mathcal{M}_{1}(\vec{Q}^{2}) - \frac{2}{3} r_{1} \mathcal{X}_{1}(\vec{Q}^{2}) \right)$$

$$- m_{s} \langle d_{3pq} D_{8p}^{(8)} D_{8q}^{(8)} \rangle_{N} \delta_{pq} \left(2\mathcal{M}_{2}(\vec{Q}^{2}) - \frac{2}{3} r_{2} \mathcal{X}_{2}(\vec{Q}^{2}) \right) \right],$$

$$(33)$$

III. RESULTS AND DISCUSSIONS

In order to evaluate Eqs. (32,33) numerically, we follow the Kahana-Ripka discretized basis method [29]. However, note that it is of great importance to use a reasonably large size of the box $(D \approx 10 \text{ fm})$ so as to get a numerically stable results. The present SU(3) χ QSM (equivalent to SU(3) NJL on the chiral circle) contains four free parameters. Two of them are fixed in the meson sector by adjusting them to the pion mass, $m_{\pi} = 139 \text{ MeV}$, the pion decay constant, $f_{\pi}=93$ MeV, and the kaon mass, $m_{K}=496$ MeV. As for the fourth parameter, i.e. the constituent mass M of up and down quarks, values around M = 420 MeV have been used because they have turned out to be the most appropriate one for the description of nucleon observables and form factors (see ref. [27]). In fact, $M=420~{\rm MeV}$ is the preferred value, which is always used in this paper. For the description of the baryon sector, we choose the method of Blotz et al. [21] modified for a finite pion mass. The resulting strange current quark mass comes out around $m_s=180$ MeV. In order to illustrate the effect of the m_s the calculations in the baryonic sector are performed with both $m_s = 0$ and finite m_s . One should note that a SU(3)-calculation with $m_s = 0$ does not correspond to a SU(2) calculation, since the spaces, in which the collective quantization are performed, are different.

Figure 1 shows the strange electric form factor $G_E^s(Q^2)$, as the constituent quark mass M is varied from 370 MeV to 450 MeV without m_s corrections. The strange electric form factor G_E^s decreases as M increases. As shown in Fig. 1, the $G_E^s(Q^2)$ without the m_s corrections is rather insensitive to the constituent quark mass M. In Fig. 2 the G_E^s with the m_s corrections is drawn. The m_s corrections enhance the G_E^s drastically, contributing to it almost 50%.

The Sachs and Dirac mean-square strange radii are, respectively, defined by

$$\langle r^2 \rangle_s^{Sachs} = -6 \frac{dG_E^s(Q^2)}{dQ^2} \mid_{Q^2=0}, \quad \langle r^2 \rangle_s^{Dirac} = -6 \frac{dF_1^s(Q^2)}{dQ^2} \mid_{Q^2=0}$$
 (34)

We obtain $\langle r^2 \rangle_s^{Sachs} = -0.25 \text{ fm}^2$ and $\langle r^2 \rangle_s^{Dirac} = -0.20 \text{ fm}^2$ without the m_s corrections in case of M = 420 MeV and $\langle r^2 \rangle_s^{Sachs} = -0.35 \text{ fm}^2$ and $\langle r^2 \rangle_s^{Dirac} = -0.32 \text{ fm}^2$ with them. As

seen from the above results, the m_s corrections increase the $\langle r^2 \rangle_s$ considerably as in case of the G_E^s . However, the mechanism of the enhancement of the $\langle r^2 \rangle_s$ by the m_s corrections is distinguished from that of the G_E^s at finite momentum transfers. Fig. 3 depicts the baryon (B) and hypercharge (Y) densities with r^2 . The m_s corrections elevate the baryon density in small r region sizably while they lessen its tail quite much in such a way that the baryon number is always kept to be one. On the other hand, the hypercharge one is almost not changed. Hence, the increase of the baryon density brings about the enhancement of the G_E^s , while the suppression of its tail increases the $\langle r^2 \rangle_s$. In fact, we obtain the baryon and hypercharge radii, respectively: $\langle r^2 \rangle_B = 0.48 \text{ fm}^2$, $\langle r^2 \rangle_Y = 0.73 \text{ fm}^2$ without the m_s corrections and $\langle r^2 \rangle_B = 0.35 \text{ fm}^2$, $\langle r^2 \rangle_Y = 0.70 \text{ fm}^2$ with them. As the m_s corrections are switched on, $\langle r^2 \rangle_B$ is almost 30% decreased and yields correspondingly the enhancement of $\langle r^2 \rangle_s$. Fig. 4 illustrates the strange electric densities weighted with r^2 . As expected from the above discussion, the m_s corrections amplify the strange electric density.

Fig. 5 draws the strange magnetic form factor without the m_s corrections. In contrast to the G_E^s , the G_M^s increases slowly with the increasing constituent quark mass apart from the small Q^2 region (below about $Q^2 = 0.2 \text{ GeV}^2$). Fig. 6 shows the G_M^s with the m_s corrections. It increases as M increases in the whole Q^2 region. As we can see from the comparison between these two figures, the G_M^s is reduced dramatically with the m_s corrections being considered. In case of M = 420 MeV, the m_s corrections bring it down almost by 40%. The strange magnetic moment is defined as the strange magnetic form factor at $Q^2 = 0$. The strange magnetic moment we have obtained is $\mu_s = -0.44 \mu_N$ in unit of the nuclear magneton. Its absolute value is rather greater than in the other models except for the recent calculation by Leinweber [17]: $\mu_s = -0.75 \pm 0.30 \mu_N$. Fig. 7 shows the strange magnetic densities with r^2 .

In table 1, the strange magnetic moments μ_s and mean-square strange radii $\langle r^2 \rangle_s$ are displayed as a function of M and m_s . In table 2, we have made a comparison for the μ_s and $\langle r^2 \rangle_s^{Sachs}$ between different models.

We want to take the occasion to comment on Ref. [32]. Though Ref. [32] seems to

use the same model as the present work, there are significant differences between these two papers. First, Weigel et al. [32] do not consider rotational $1/N_c$ corrections in contrast to the present paper. This has the immediate consequence that the magnetic moments of Weigel et al. are $\mu_p = 1.06 \ \mu_N$, $\mu_n = -0.69 \ \mu_N$ for the nucleon ¹ whereas the present work (including those corrections) yields $\mu_p = 2.20 \ \mu_N$, $\mu_n = -1.59 \ \mu_N$ with a far better comparison with experiment ($\mu_p = 2.79 \ \mu_N$, $\mu_n = -1.91 \ \mu_N$). Furthermore, Weigel et al. regularize, besides the real part of the action, also the imaginary one. This meets problems in producing the anomaly structure and is hence avoided in the approach of the present work. In addition the calculation of Weigel et al. are not fully self-consistent but use some scaling approximations.

IV. SUMMARY AND CONCLUSION

In summary, we have calculated in the SU(3) chiral quark-soliton model (χ QSM) often called the semibosonized SU(3) Nambu–Jona-Lasinio model, the strange electric and magnetic form factors of the nucleon, $G_E^s(Q^2)$ and $G_M^s(Q^2)$ including the strange magnetic moment μ_s , and the mean-square strange radius $\langle r^2 \rangle_s$. The theory takes into account rotational $1/N_c$ corrections and linear m_s corrections. We have obtained $\mu_s = -0.44 \ \mu_N$, $\langle r^2 \rangle_s^{Dirac} = -0.32 \ \text{fm}^2$ and $\langle r^2 \rangle_s^{Sachs} = -0.35 \ \text{fm}^2$. The results have been compared with different other models.

There are several points where the present calculations leave room for further studies. Apparently the dependence of the form factors on the value of m_s is quite noticeable and probably one has to go to higher orders in perturbation theory in m_s . The hedgehog ansatz and the embedding of SU(2) into SU(3) cause the asymptotic behavior of the kaon and pion fields to be similar, which might have some influence on the form factors at low momentum transfers. Besides the strange vector form factors the strange axial form factors are also of

¹For this comparison, the constituent quark mass M=450 MeV is chosen. In case of M=420 MeV, we have obtained $\mu_p=2.39$ μ_N and $\mu_n=-1.76$ μ_N [24].

great interest. Presently we are performing investigations to clarify these questions.

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APPENDIX A:

In this appendix, we present all formulae appearing in Eqs. (32,33).

with regularization functions

(A3)

$$\mathcal{M}_{2}(\vec{Q}^{2}) = \frac{N_{c}}{3} \sum_{m^{0}} \int d^{3}x j_{1}(qr) \int d^{3}y$$

$$\times \left[\frac{\Psi_{m^{0}}^{\dagger}(x) \gamma_{5} \{\hat{r} \times \vec{\sigma}\} \cdot \vec{\tau} \Psi_{val}(x) \Psi_{val}^{\dagger}(y) \beta \Psi_{m^{0}}(y)}{E_{m^{0}} - E_{val}} + \sum_{n} \Psi_{n}^{\dagger}(x) \gamma_{5} \{\hat{r} \times \vec{\sigma}\} \cdot \vec{\tau} \Psi_{m^{0}}(x) \Psi_{m^{0}}^{\dagger}(y) \beta \Psi_{n}(y) \mathcal{R}_{\beta}(E_{n}, E_{m^{0}}) \right]. \tag{A4}$$

The regularization functions for the ${\cal G}_M^s$ are

$$\mathcal{R}(E_n) = \int \frac{du}{\sqrt{\pi u}} \phi(u; \Lambda_i) |E_n| e^{-uE_n^2},$$

$$\mathcal{R}_{\mathcal{Q}}(E_n, E_m) = \frac{1}{2\pi} c_i \int_0^1 d\alpha \frac{\alpha(E_n + E_m) - E_m}{\sqrt{\alpha(1 - \alpha)}} \frac{\exp\left(-\left[\alpha E_n^2 + (1 - \alpha)E_m^2\right]/\Lambda_i^2\right)}{\alpha E_n^2 + (1 - \alpha)E_m^2},$$

$$\mathcal{R}_{\beta}(E_n, E_m) = \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{du}{\sqrt{u}} \phi(u; \Lambda_i) \left[\frac{E_n e^{-uE_n^2} - E_m e^{-uE_m^2}}{E_n - E_m}\right].$$
(A5)

The cutoff parameter $\phi(u; \Lambda_i) = \sum_i c_i \theta\left(u - \frac{1}{\Lambda_i^2}\right)$ is fixed by reproducing the pion decay constants and other mesonic properties [27].

TABLES

TABLE I. The strange magnetic moments and mean-square strange radius as varying the constituent quark mass.

M	370	MeV	420	MeV	450	MeV
$m_s \; [{ m MeV}]$	0	180	0	180	0	180
$\mu_s[\mu_N]$	-0.87	-0.37	-0.78	-0.44	-0.74	-0.50
$\langle r^2\rangle_s^{Dirac} [{\rm fm}^2]$	-0.22	-0.47	-0.19	-0.32	-0.16	-0.27
$\langle r^2\rangle_s^{Sachs} [\rm fm^2]$	-0.28	-0.49	-0.25	-0.35	-0.21	-0.31

TABLE II. The theoretical comparison for the strange magnetic moment and mean-square strange radius between different models. $M=420~{\rm MeV}$ is used for the present work.

models	$\mu_s[\mu_N]$	$\langle r^2 \rangle_s^{Sachs} [\mathrm{fm}^2]$	references
Jaffe	-0.31 ± 0.09	0.14 ± 0.07	[9]
Hammer et al.	-0.24 ± 0.03	0.23 ± 0.03	[11]
Koepf et al.	-2.6×10^{-2}	-0.97×10^{-2}	[12]
Musolf & Burkhardt	$-(0.31 \to 0.40)$	$-(2.71 \rightarrow 3.23) \times 10^{-2}$	[13]
Cohen et al.	$-(0.24 \to 0.32)$	$-(3.99 \to 4.51) \times 10^{-2}$	[14]
Forkel et al.		1.69×10^{-2}	[18]
Park & Weigel	-0.05	0.05	[16]
Park et al.	-0.13	-0.11	[15]
Leinweber	-0.75 ± 0.30		[17]
Alberico et al.	-0.14	0.055	[31]
Weigel et al.	$-0.05 \rightarrow 0.25$	$-0.2 \rightarrow -0.1$	[32]
$SU(3) \chi QSM$	-0.44	-0.35	Present work

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Figure Captions

- Fig. 1: The strange electric form factor G_E^s as functions of Q^2 without the m_s corrections: The solid curve corresponds to the constituent quark mass M=420 MeV, while dot-dashed curve draws M=370 MeV. The dashed curve displays the case of M=450 MeV. The M=420 MeV is distinguished since all other observables of the nucleon are then basically reproduced in this model.
- Fig. 2: The strange electric form factor G_E^s as functions of Q^2 with $m_s = 180$ MeV: The solid curve corresponds to the constituent quark mass M=420 MeV, while dot-dashed curve draws M=370 MeV. The dashed curve displays the case of M=450 MeV. The M=420 MeV is distinguished since all other observables of the nucleon are then basically reproduced in this model.
- Fig. 3: The baryon and hypercharge densities as functions of r. The solid curve draws the baryon charge density with $m_s = 180$ MeV while the dashed one for the same density without the m_s corrections. The dot-dashed curve designates the hypercharge density with $m_s = 180$ MeV while the dot-dot-dashed one for the same density without the m_s corrections. The constituent quark mass M = 420 MeV is used.
- Fig. 4: The strange electric density $r^2 \rho_E^s$ as functions of r: The solid curve draws the $r^2 \rho_E^s$ with $m_s = 180$ MeV, whereas the dashed one displays it without the m_s corrections. The constituent quark mass M = 420 MeV is used.
- Fig. 5: The strange magnetic form factor without the m_s corrections as a function of Q^2 : The solid curve corresponds to the constituent quark mass M=420 MeV, while dashed curve draws the case of M=450 MeV. The dot-dashed curve displays the case of M=370 MeV. The strange quark mass is taken to be $m_s=0$. The M=420 MeV is distinguished

since all other observables of the nucleon are then basically reproduced in this model.

Fig. 6: The strange magnetic form factor with $m_s = 180$ MeV as a function of Q^2 : The solid curve corresponds to the constituent quark mass M = 420 MeV, while dashed curve draws the case of M = 450 MeV. The dot-dashed curve displays the case of M = 370 MeV. The strange quark mass is taken to be $m_s = 180$ MeV. The M=420 MeV is distinguished since all other observables of the nucleon are then basically reproduced in this model.

Fig. 7: The strange magnetic density $r^2 \rho_M^s$ as functions of r: The solid curve draws the $r^2 \rho_M^s$ with $m_s = 180$ MeV, whereas the dashed one displays it without the m_s corrections. The constituent quark mass M = 420 MeV is used.

Figures

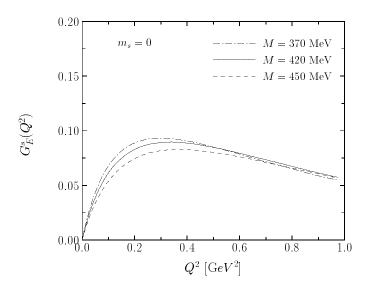


Figure 1

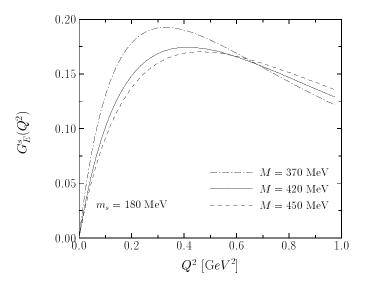


Figure 2

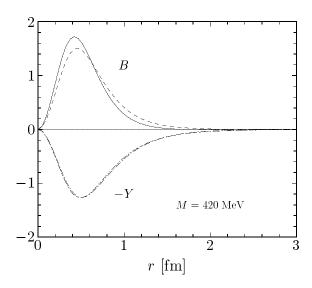


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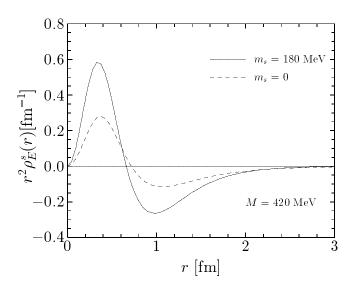


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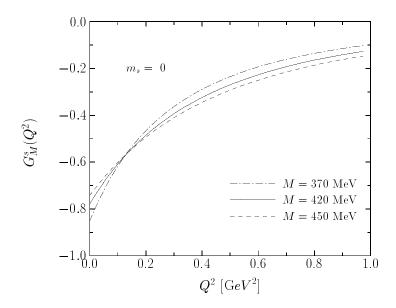


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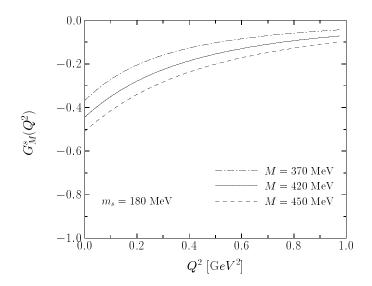


Figure 6

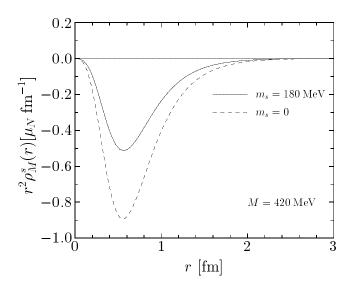


Figure 7